Scalar sector of Supersymmetric $SU(3)_C \otimes SU(3)_L \otimes U(1)_N$ Model with right-handed neutrinos

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Abstract

We investigate a scalar sector of the supersymmetric $SU_C(3) \otimes SU_L(3) \otimes U_N(1)$ model with right-handed neutrinos. The mass spectra are derived. We show that only neutral Higgs sector with lepton number $L = 0$ could have a VEV. There is no mixing between scalars having $L = 0$ and bilepton scalars having $L = 2$. There are six Goldstone bosons: two in neutral sector, three in pseudo-scalar sector and one in charged scalar sector. For a given set of input parameters (five from the F terms and two from the soft term) all the scalar sectors in this model contain the upper limit of 230 GeV to the mass of the lightest scalar, which are in agreement with the lower limit of the SM Higgs boson obtained by LEP.

1 Introduction

The models based on the $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ (called 3-3-1 models) [1, 2, 3] provide possible solutions to some puzzles of the standard model (SM) such as the generation number problem, the electric charge quantization [4]. Since one generation of quarks is treated differently from the others this may lead to a natural explanation for the large mass of the top quark [5]. These models also furnish a good candidate for self-interacting dark matter (SIDM) since there are two Higgs bosons, one scalar and one pseudoscalar, which have the properties of candidates for dark matter like stability, neutrality and that it must not overpopulate the universe [6], etc.

There are two main versions of the 3-3-1 models as far as lepton sector is concern. In the minimal version, the charge conjugation of the right-handed charged lepton for each

generation is combined with the usual $SU(2)_L$ doublet left-handed leptons components to form an $SU(3)$ triplet $(\nu, l, l^c)_L$. No extra leptons are needed and there we shall call such models minimal 3-3-1 models. There is no right-handed (RH) neutrino in its minimal version. Another version adds a left-handed anti-neutrino to each usual $SU(2)_L$ doublet left-handed lepton to form a triplet. This model is called the 3-3-1 model with RH neutrinos.

The supersymmetric version of the model of Ref.[3] has already been presented in Ref.[7]. However, the authors of Ref.[7] have just mentioned on the neutral Higgs sector. The Higgs sector still remains one of the most indefinite part of the SM, but it still represents a fundamental rule by explaining how the particles gain masses by means of an isodoublets scalar field, which is responsible for the spontaneous breakdown of the gauge symmetry, the process by which the spectra of all particles are generated.The Higgs mechanism plays a central role in gauge theories. In this paper we will consider in detail the Higgs sector of the model.

This paper is organized as follows. In Sec. 2 we review the non-supersymmetric 3-3-1 model with RH neutrinos and introduce the respective superpartners. The construction of the supersymmetric scalar potential is discussed in Sec. 3, while in Sec. 4 we derive the mass spectrum of the scalar sector of the model. The numerical analysis are given in Sec. 5 and our plots are presented in Sec. 6. Finally, the last section is devoted to our conclusions.

2   A review of the model

In this section we present one review of the model we will consider on this work.

2.1   The non-supersymmetric 331 model with RH neutrinos

Let us first summarize the non-supersymmetric model [3]. The leptons transforming under the 3-3-1 factors as

$$L_{aL} = \begin{pmatrix} \nu_a \\ l_a \\ \nu_a^c \end{pmatrix}_L \sim (\mathbf{1}, \mathbf{3}, -1/3), \tag{2.1}$$

with $a = e, \mu, \tau$ and $\nu_a^c = C\bar{\nu}_a^{\,T}$, plus the singlets

$$l_{aL}^c \sim (\mathbf{1}, \mathbf{1}, 1). \tag{2.2}$$

In the quark sector we have the first two families transforming as antitriplets of $SU(3)_L$

$$Q_{\alpha L} = \begin{pmatrix} d_\alpha \\ u_\alpha \\ D_\alpha \end{pmatrix}_L \sim (\mathbf{3}, \mathbf{3}^*, 0), \quad \alpha = 1, 2; \tag{2.3}$$

with the respective singlets

$$u_{\alpha L}^c \sim (\mathbf{3}^*, \mathbf{1}, -2/3), \quad d_{\alpha L}^c, \ D_{\alpha L}^c \sim (\mathbf{3}^*, \mathbf{1}, 1/3). \tag{2.4}$$

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The third family transforms as triplet under $SU(3)_L$

$$Q_{3L} = \begin{pmatrix} u_3 \\ d_3 \\ T \end{pmatrix}_L \sim (\mathbf{3}, \mathbf{3}, 1/3), \tag{2.5}$$

and their respective singlets

$$u_{3L}^c, \ T_L^c \sim (\mathbf{3}^*, \mathbf{1}, -2/3), \ \ d_{3L}^c \sim (\mathbf{3}^*, \mathbf{1}, 1/3). \tag{2.6}$$

In the scalar sector only two triplets $\eta \sim (\mathbf{1}, \mathbf{3}, -1/3)$ and $\rho \sim (\mathbf{1}, \mathbf{3}, 2/3)$ are necessary to break appropriately the gauge symmetry and also to give the correct mass to all the fermions in the model. However, to eliminate flavor changing neutral currents we add an extra scalar triplet transforming like η. Therefore, the scalars of our model are written as

$$\eta = \begin{pmatrix} \eta_1^0 \\ \eta^- \\ \eta_2^0 \end{pmatrix}, \quad \chi = \begin{pmatrix} \chi_1^0 \\ \chi^- \\ \chi_2^0 \end{pmatrix} \sim (\mathbf{1}, \mathbf{3}, -1/3),$$

$$\rho = \begin{pmatrix} \rho_1^+ \\ \rho^0 \\ \rho_2^+ \end{pmatrix} \sim (\mathbf{1}, \mathbf{3}, 2/3), \tag{2.7}$$

and we will denote the vacuum expectation values which are different from zero as $v = \langle \eta_1^0 \rangle$, $w = \langle \chi_2^0 \rangle$ and $u = \langle \rho^0 \rangle$.

Despite η and χ have the same quantum number, but they members are quite different [8]:

$$L(\eta_1^0, \eta^-, \rho_1^+, \rho^0, \chi_2^0) = 0,$$
$$L(\eta_2^0, \rho_2^+, \chi_1^0, \chi^-) = 2 \tag{2.8}$$

The η_2^0 and χ_1^0 are scalar bileptons $L(\eta_2^0, \chi_1^0) = 2$, while ρ^0, χ_2^0 do not have lepton number $L(\rho^0, \chi_2^0) = 0$ [8]. It is to be noted that, only pure (*without lepton number*) neutral scalars can have VEVs.

With this mention, one expect that the would-be Goldstone bosons coming from the Higgs potential can be written as:

$$\rho = \begin{pmatrix} G_W^+ \\ u + iG_Z \\ 0 \end{pmatrix}, \quad \eta = \begin{pmatrix} v + iG_Z \\ -G_W^- \\ 0 \end{pmatrix}, \quad \chi = \begin{pmatrix} G_X^0 \\ G_Y^- \\ w + iG_{Z'} \end{pmatrix} \tag{2.9}$$

where $G_W^\pm$, G_Z, $G_Y^\pm$ and G_X^0, G_X^{0*} are the would-be Goldstone bosons for the fields $W^\pm$, Z, $Y^\pm$ and X^0, X^{0*}, respectively. The ρ and η components give origin to the charged Higgs H^+, odd-A^0, even-H^0 (all with masses of the order of w, scale of energy of the first symmetry breaking) and the light Higgs h^0 coming from the electroweak scale. The Higgs fields ρ_2^+, η_2^0 and χ_2^0 have a mass proportional to the scale w. The other fields of χ give origin to the would-be Goldstone bosons of X^0, Y^- and Z'. All the scalar fields, except for h^0, have masses of the order of the first symmetry breaking M_χ.

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2.2 Supersymmetric partners

Now, we introduce the minimal set of particles in order to implement the supersymmetry. Here we will follow the usual notation writing for a given fermion f, the respective sfermions by $\tilde{f}$ *i.e.*, $\tilde{l}$ and $\tilde{q}$ denote sleptons and squarks respectively [9]. Then, we have the following additional particles

$$
\tilde{Q}_{\alpha L} = \begin{pmatrix} \tilde{d}_\alpha \\ \tilde{u}_\alpha \\ \tilde{D}_\alpha \end{pmatrix}_L \sim (\mathbf{3}, \mathbf{3}^*, 0), \ \tilde{Q}_{3L} = \begin{pmatrix} \tilde{u}_3 \\ \tilde{d}_3 \\ \tilde{T} \end{pmatrix}_L \sim (\mathbf{3}, \mathbf{3}, 1/3),
$$

$$
\tilde{L}_{aL} = \begin{pmatrix} \tilde{\nu}_a \\ \tilde{l}_a \\ \tilde{\nu}_a^c \end{pmatrix}_L \sim (\mathbf{1}, \mathbf{3}, -1/3), \tag{2.10}
$$

$$
\tilde{l}_{aL}^c \sim (\mathbf{1}, \mathbf{1}, 1),
$$
$$
\tilde{u}_{iL}^c, \tilde{T}_L^c \sim (\mathbf{3}^*, \mathbf{1}, -2/3), \ \tilde{d}_{iL}^c, \tilde{D}_{\alpha L}^c \sim (\mathbf{3}^*, \mathbf{1}, 1/3), \tag{2.11}
$$

with $a = e, \mu, \tau$; $i = 1, 2, 3$; and $\alpha = 1, 2$. However, when considering quark (or squark) singlets of a given charge we will use the notation u_{iL}^c, d_{iL}^c ($\tilde{u}_{iL}, \tilde{d}_{iL}^c$ with $i(j) = 1, 2, 3$).

The supersymmetric partner of the scalar Higgs fields, the higgsinos, are

$$
\tilde{\eta} = \begin{pmatrix} \tilde{\eta}_1^0 \\ \tilde{\eta}^- \\ \tilde{\eta}_2^0 \end{pmatrix}, \ \tilde{\chi} = \begin{pmatrix} \tilde{\chi}_1^0 \\ \tilde{\chi}^- \\ \tilde{\chi}_2^0 \end{pmatrix} \sim (\mathbf{1}, \mathbf{3}, -1/3),
$$

$$
\tilde{\rho} = \begin{pmatrix} \tilde{\rho}_1^+ \\ \tilde{\rho}^0 \\ \tilde{\rho}_2^+ \end{pmatrix} \sim (\mathbf{1}, \mathbf{3}, 2/3), \tag{2.12}
$$

and the respective extra higgsinos, needed to cancel the chiral anomaly of the higgsinos in Eq. (2.12), are

$$
\tilde{\eta}' = \begin{pmatrix} \tilde{\eta}_1'^0 \\ \tilde{\eta}'^+ \\ \tilde{\eta}_2'^0 \end{pmatrix}, \tilde{\chi}' = \begin{pmatrix} \tilde{\chi}_1'^0 \\ \tilde{\chi}'^+ \\ \tilde{\chi}_2'^0 \end{pmatrix} \sim (\mathbf{1}, \mathbf{3}^*, 1/3),
$$

$$
\tilde{\rho}' = \begin{pmatrix} \tilde{\rho}_1'^- \\ \tilde{\rho}'^0 \\ \tilde{\rho}_2'^- \end{pmatrix} \sim (\mathbf{1}, \mathbf{3}^*, -2/3), \tag{2.13}
$$

and the corresponding scalar partners denoted by η', χ', ρ', with the same charge assignment as in Eq. (2.13), and with the following VEVs: $v' = \langle \eta_1'^0 \rangle$, $w' = \langle \chi_2'^0 \rangle$ and $u' = \langle \rho'^0 \rangle$.

Concerning the gauge bosons and their superpartners, if we denote the gluons by g^b the respective superparticles, the gluinos, are denoted by λ_C^b, with $b = 1, \ldots, 8$; and in the electroweak sector we have V^b, the gauge boson of $SU(3)_L$, and their gauginos partners λ_A^b; finally we have the gauge boson of $U(1)_N$, denoted by V', and its supersymmetric partner λ_B. This is the total number of fields in the minimal supersymmetric extension of the 3-3-1 model of Refs. [3].

3 The supersymmetric scalar potential and mass spectrum

It is well known that in supersymmetric models, that the contributions to the scalar potential arise from three sources - the auxiliary F- and D- fields [10] and the soft terms [9].

On this article we will write only necessary terms to pick all the terms needed to construct the scalar potential of our model, see [7] to the complete lagrangian.

3.1 Elimination of the auxiliary fields

The lagrangian of the gauge sector is a source of the D- terms and it is written as

$$
\begin{aligned}
\mathcal{L}^{gauge} &= \frac{1}{4} \int d^2\theta \, Tr[\mathcal{W}_\mathcal{C}\mathcal{W}_\mathcal{C}] + \frac{1}{4} \int d^2\theta \, Tr[\mathcal{W}_\mathcal{L}\mathcal{W}_\mathcal{L}] + \frac{1}{4} \int d^2\theta \mathcal{W}'\mathcal{W}' \\
&+ \frac{1}{4} \int d^2\bar{\theta} \, Tr[\bar{\mathcal{W}}_\mathcal{C}\bar{\mathcal{W}}_\mathcal{C}] + \frac{1}{4} \int d^2\bar{\theta} \, Tr[\bar{\mathcal{W}}_\mathcal{L}\bar{\mathcal{W}}_\mathcal{L}] + \frac{1}{4} \int d^2\bar{\theta}\bar{\mathcal{W}}'\bar{\mathcal{W}}' \,,
\end{aligned}
\tag{3.1}
$$

where $\mathcal{W}_\mathcal{C}$, $\mathcal{W}$ and $\mathcal{W}'$ are fields that can be written as follows

$$
\begin{aligned}
\mathcal{W}_{\zeta\mathcal{C}} &= -\frac{1}{8g_s}\bar{D}\bar{D}e^{-2g_s\hat{V}_C}D_\zeta e^{2g_s\hat{V}_C}, \\
\mathcal{W}_{\zeta\mathcal{L}} &= -\frac{1}{8g}\bar{D}\bar{D}e^{-2g\hat{V}}D_\zeta e^{2g\hat{V}}, \\
\mathcal{W}'_\zeta &= -\frac{1}{4}\bar{D}\bar{D}D_\zeta\hat{V}', \ \ \zeta = 1, 2.
\end{aligned}
\tag{3.2}
$$

The coupling g_s is the gauge coupling constants of $SU(3)_c$ while g and g' are the gauge coupling constants of $SU(3)_L$ and $U(1)_N$, respectively.

In the scalar sector, both F- and D- terms yield the following lagrangian

$$
\begin{aligned}
\mathcal{L}^{scalar} &= \int d^4\theta \, \left[\hat{\bar{\eta}} e^{[2g\hat{V} + g'\left(-\frac{1}{3}\right)\hat{V}']}\hat{\eta} + \hat{\bar{\chi}} e^{[2g\hat{V} + g'\left(-\frac{1}{3}\right)\hat{V}']}\hat{\chi} + \hat{\bar{\rho}} e^{[2g\hat{V} + g'\left(\frac{2}{3}\right)\hat{V}']}\hat{\rho} \right. \\
&+ \left. \hat{\bar{\eta}}' e^{[2g\hat{V} + g'\left(\frac{1}{3}\right)\hat{V}']}\hat{\eta}' + \hat{\bar{\chi}}' e^{[2g\hat{V} + g'\left(\frac{1}{3}\right)\hat{V}']}\hat{\chi}' + \hat{\bar{\rho}}' e^{[2g\hat{V} + g'\left(-\frac{2}{3}\right)\hat{V}']}\hat{\rho}' \right] \\
&+ \int d^2\theta \, W + \int d^2\bar{\theta} \, \overline{W}.
\end{aligned}
\tag{3.3}
$$

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W is the superpotential of the model, it is only F- term source. The superpotential is decomposed as follows

$$W = \frac{W_2}{2} + \frac{W_3}{3} \tag{3.4}$$

and it can be written explicitly as

$$
\begin{aligned}
W_2 &= \mu_{0a}\hat{L}_a\hat{\eta}' + \mu_{1a}\hat{L}_a\hat{\chi}' + \mu_\eta\hat{\eta}\hat{\eta}' + \mu_\chi\hat{\chi}\hat{\chi}' + \mu_\rho\hat{\rho}\hat{\rho}', \\
W_3 &= \lambda_{1ab}\hat{L}_a\hat{\rho}'\hat{l}_b^c + \lambda_{2a}\epsilon\hat{L}_a\hat{\chi}\hat{\rho} + \lambda_{3a}\epsilon\hat{L}_a\hat{\eta}\hat{\rho} + \lambda_{4ab}\epsilon\hat{L}_a\hat{L}_b\hat{\rho} + \kappa_{1i}\hat{Q}_3\hat{\eta}'\hat{u}_i^c + \kappa_1'\hat{Q}_3\hat{\eta}'\hat{u}'^c \\
&+ \kappa_{2i}\hat{Q}_3\hat{\chi}'\hat{u}_i^c + \kappa_2'\hat{Q}_3\hat{\chi}'\hat{u}'^c + \kappa_{3\alpha i}\hat{Q}_\alpha\hat{\eta}\hat{d}_i^c + \kappa'_{3\alpha\beta}\hat{Q}_\alpha\hat{\eta}\hat{d}'^c_\beta + \kappa_{4\alpha i}\hat{Q}_\alpha\hat{\rho}\hat{u}_i^c + \kappa'_{4\alpha}\hat{Q}_\alpha\hat{\rho}\hat{u}'^c \\
&+ \kappa_{5i}\hat{Q}_3\hat{\rho}'\hat{d}_i^c + \kappa'_{5\beta}\hat{Q}_3\hat{\rho}'\hat{d}'^c_\beta + \kappa_{6\alpha i}\hat{Q}_\alpha\hat{\chi}\hat{d}_i^c + \kappa'_{6\alpha\beta}\hat{Q}_\alpha\hat{\chi}\hat{d}'^c_\beta + f_1\epsilon\hat{\rho}\hat{\chi}\hat{\eta} + f_1'\epsilon\hat{\rho}'\hat{\chi}'\hat{\eta}' \\
&+ \zeta_{\alpha\beta\gamma}\epsilon\hat{Q}_\alpha\hat{Q}_\beta\hat{Q}_\gamma + \lambda'_{\alpha ai}\hat{Q}_\alpha\hat{L}_a\hat{d}_i^c + \lambda''_{ijk}\hat{d}_i^c\hat{u}_j^c\hat{d}_k^c + \xi_{1ij\beta}\hat{d}_i^c\hat{u}_j^c\hat{d}'^c_\beta + \xi_{2\alpha a\beta}\hat{Q}_\alpha\hat{L}_a\hat{d}'^c_\beta \\
&+ \xi_{3i\beta}\hat{d}_i^c\hat{u}'^c\hat{d}'^c_\beta + \xi_{4ij}\hat{d}_i^c\hat{u}'^c\hat{d}_j^c + \xi_{5\alpha i\beta}\hat{d}'^c_\alpha\hat{u}_i^c\hat{d}'^c_\beta + \xi_{6\alpha\beta}\hat{d}'^c_\alpha\hat{u}'^c\hat{d}'^c_\beta
\end{aligned} \tag{3.5}
$$

The coefficients $\mu_0, \mu_1, \mu_\eta, \mu_\rho$ and μ_χ have mass dimension, while all the coefficients in W_3 are dimensionless.

To get the scalar potential of our model we have to pick up the F and D- terms, from Eqs. (3.1,3.3,3.5) and get

$$
\begin{aligned}
\mathcal{L}_F &= \mathcal{L}_F^{scalar} + \mathcal{L}_F^{W2} + \mathcal{L}_F^{W3} \\
&= |F_\eta|^2 + |F_\rho|^2 + |F_\chi|^2 + |F_{\eta'}|^2 + |F_{\rho'}|^2 + |F_{\chi'}|^2 \\
&+ \frac{\mu_\eta}{2}(\eta F_{\eta'} + \eta' F_\eta + \eta^\dagger F_{\eta'}^\dagger + \eta'^\dagger F_\eta^\dagger) + \frac{\mu_\rho}{2}(\rho F_{\rho'} + \rho' F_\rho + \rho^\dagger F_{\rho'}^\dagger + \rho'^\dagger F_\rho^\dagger) \\
&+ \frac{\mu_\rho}{2}(\rho F_{\rho'} + \rho' F_\rho + \rho^\dagger F_{\rho'}^\dagger + \rho'^\dagger F_\rho^\dagger) + \frac{1}{3}\left[f_1\epsilon(F_\rho\chi\eta + \rho F_\chi\eta + \rho\chi F_\eta\right. \\
&+ F_\rho^\dagger\chi^\dagger\eta^\dagger + \rho^\dagger F_\chi^\dagger\eta^\dagger + \rho^\dagger\chi^\dagger F_\eta^\dagger) + f_1'\epsilon(F_{\rho'}\chi'\eta' + \rho' F_{\chi'}\eta' + \rho'\chi' F_{\eta'} \\
&+ \left. F_{\rho'}^\dagger\chi'^\dagger\eta'^\dagger + \rho'^\dagger F_{\chi'}^\dagger\eta'^\dagger + \rho'^\dagger\chi'^\dagger F_{\eta'}^\dagger)\right] \; , \\
\mathcal{L}_D &= \mathcal{L}_D^{gauge} + \mathcal{L}_D^{scalar} \\
&= \frac{1}{2}D^a D^a + \frac{1}{2}DD + \frac{g}{2}\left[\eta^\dagger\lambda^a\eta + \rho^\dagger\lambda^a\rho + \chi^\dagger\lambda^a\chi - \eta'^\dagger\lambda^{*a}\eta' - \rho'^\dagger\lambda^{*a}\rho'\right. \\
&- \left.\chi'^\dagger\lambda^{*a}\chi'\right]D^a + \frac{g'}{2}\left[-\frac{1}{3}\eta^\dagger\eta + \frac{1}{3}\eta'^\dagger\eta' - \frac{1}{3}\chi^\dagger\chi + \frac{1}{3}\chi'^\dagger\chi' + \frac{2}{3}\rho^\dagger\rho - \frac{2}{3}\rho'^\dagger\rho'\right]D.
\end{aligned} \tag{3.6}
$$

We will now show that these fields can be eliminated through the Euler-Lagrange equations

$$\frac{\partial\mathcal{L}}{\partial\phi} - \partial_m\frac{\partial\mathcal{L}}{\partial(\partial_m\phi)} = 0 \; , \tag{3.7}$$

where $\phi = \eta, \rho, \chi, \eta', \rho', \chi'$. Formally auxiliary fields are defined as fields having no kinetic terms. Thus, this definition immediately yields that the Euler-Lagrange equations for auxiliary fields simplify to $\frac{\partial\mathcal{L}}{\partial\phi} = 0$.

Applying these simplified equations to various auxiliary F-fields yields the following relations

$$F_\eta^\dagger = -\left(\frac{\mu_\eta}{2}\eta' + \frac{f_1}{3}\epsilon\rho\chi\right) \;\; ; \;\; F_\eta = -\left(\frac{\mu_\eta}{2}\eta'^\dagger + \frac{f_1}{3}\epsilon\rho^\dagger\chi^\dagger\right) \;,$$

$$F_\rho^\dagger = -\left(\frac{\mu_\rho}{2}\rho' + \frac{f_1}{3}\epsilon\chi\eta\right) \;\; ; \;\; F_\rho = -\left(\frac{\mu_\rho}{2}\rho'^\dagger + \frac{f_1}{3}\epsilon\chi^\dagger\eta^\dagger\right) \;,$$

$$F_\chi^\dagger = -\left(\frac{\mu_\chi}{2}\chi' + \frac{f_1}{3}\epsilon\rho\eta\right) \;\; ; \;\; F_\chi = -\left(\frac{\mu_\chi}{2}\chi'^\dagger + \frac{f_1}{3}\epsilon\rho^\dagger\eta^\dagger\right) \;,$$

$$F_{\eta'}^\dagger = -\left(\frac{\mu_\eta}{2}\eta + \frac{f_1'}{3}\epsilon\rho'\chi'\right) \;\; ; \;\; F_{\eta'} = -\left(\frac{\mu_\eta}{2}\eta^\dagger + \frac{f_1'}{3}\epsilon\rho'^\dagger\chi'^\dagger\right) \;,$$

$$F_{\rho'}^\dagger = -\left(\frac{\mu_\rho}{2}\rho + \frac{f_1'}{3}\epsilon\chi'\eta'\right) \;\; ; \;\; F_{\rho'} = -\left(\frac{\mu_\rho}{2}\rho^\dagger + \frac{f_1'}{3}\epsilon\chi'^\dagger\eta'^\dagger\right) \;,$$

$$F_{\chi'}^\dagger = -\left(\frac{\mu_\chi}{2}\chi + \frac{f_1'}{3}\epsilon\rho'\eta'\right) \;\; ; \;\; F_{\chi'} = -\left(\frac{\mu_\chi}{2}\chi^\dagger + \frac{f_1'}{3}\epsilon\rho'^\dagger\eta'^\dagger\right) \;. \tag{3.8}$$

Using these equations, we can rewrite Eq.(3.6) as

$$\mathcal{L}_F = -\left(|F_\eta|^2 + |F_\rho|^2 + |F_\chi|^2 + |F_{\eta'}|^2 + |F_{\rho'}|^2 + |F_{\chi'}|^2\right). \tag{3.9}$$

Performing the same program to D-fields we get

$$D^a = -\frac{g}{2}\left[\eta^\dagger\lambda^a\eta + \rho^\dagger\lambda^a\rho + \chi^\dagger\lambda^a\chi - \eta'^\dagger\lambda^{*a}\eta' - \rho'^\dagger\lambda^{*a}\rho' - \chi'^\dagger\lambda^{*a}\chi'\right] \;,$$

$$D = -\frac{g'}{2}\left[-\frac{1}{3}\eta^\dagger\eta + \frac{1}{3}\eta'^\dagger\eta' - \frac{1}{3}\chi^\dagger\chi + \frac{1}{3}\chi'^\dagger\chi' + \frac{2}{3}\rho^\dagger\rho - \frac{2}{3}\rho'^\dagger\rho'\right] \;, \tag{3.10}$$

which is in accordance with Eq.(3.6)

$$\mathcal{L}_D = -\frac{1}{2}\left(D^aD^a + DD\right). \tag{3.11}$$

3.2   The soft term

Now we are considering the last source to construct our scalar potential. The most general soft supersymmetry breaking terms, which do not induce quadratic divergence, where described by Girardello and Grisaru [11]. They found that the allowed terms can be categorized as follows: a scalar field A with mass terms

$$-m^2 A^\dagger A, \tag{3.12}$$

a fermion field gaugino λ with mass terms

$$-\frac{1}{2}(M_\lambda\lambda^a\lambda^a + H.c) \tag{3.13}$$

and finally trilinear scalar interaction terms

$$\epsilon^{ijk} A_i A_j A_k + H.c. \tag{3.14}$$

The terms on this case are similar with the terms allowed in the superpotential of the model we are considering.

Of course, the form of these terms, depend on the model we are considering, in our case see [7]. The only necessary part for us on this article is given by

$$
\begin{aligned}
\mathcal{L}_{\text{scalar}}^{\text{soft}} &= -m_\eta^2 \eta^\dagger \eta - m_\rho^2 \rho^\dagger \rho - m_\chi^2 \chi^\dagger \chi - m_{\eta'}^2 \eta'^\dagger \eta' - m_{\rho'}^2 \rho'^\dagger \rho' - m_{\chi'}^2 \chi'^\dagger \chi' \\
&+ [k_1 \epsilon_{ijk} \rho_i \chi_j \eta_k + k_1' \epsilon_{ijk} \rho_i' \chi_j' \eta_k' + H.c.].
\end{aligned} \tag{3.15}
$$

4 The supersymmetric scalar potential and mass spectrum

The pattern of the symmetry breaking in this model is given by the following scheme

$$
\begin{aligned}
\text{MSUSY331} \;&\overset{\mathcal{L}_{soft}}{\longmapsto}\; SU(3)_C \;\otimes\; SU(3)_L \otimes U(1)_N \;\overset{\langle\chi\rangle\langle\chi'\rangle}{\longmapsto}\; SU(3)_C \;\otimes\; SU(2)_L \otimes U(1)_Y \\
&\overset{\langle\rho,\eta,\rho'\eta'\rangle}{\longmapsto}\; SU(3)_C \;\otimes\; U(1)_Q
\end{aligned} \tag{4.1}
$$

In the considered model, we have three extra triplets, therefore in the neutral scalar sector we have 10×10 matrix instead of 5×5 in the non-supersymmetric gauge theory $SU_c(3) \otimes SU_L(3) \otimes U_N(1)$ [6, 12]. As mentioned in [7], the supersymmetric Higgs potential can be written:

$$V_{331SUSYRN} = V_F + V_D + V_{\text{soft}}, \tag{4.2}$$

where

$$
\begin{aligned}
V_F &= -\mathcal{L}_F = \sum_m F_m^\dagger F_m \\
&= \sum_{ijk} \Bigg[\left| \frac{\mu_\eta}{2} \eta_i' + \frac{f_1}{3} \epsilon_{ijk} \rho_j \chi_k \right|^2 + \left| \frac{\mu_\chi}{2} \chi_i' + \frac{f_1}{3} \epsilon_{ijk} \eta_j \rho_k \right|^2 + \left| \frac{\mu_\rho}{2} \rho_i' + \frac{f_1}{3} \epsilon_{ijk} \chi_j \eta_k \right|^2 \\
&+ \left| \frac{\mu_\eta}{2} \eta_i + \frac{f_1'}{3} \epsilon_{ijk} \rho_j' \chi_k' \right|^2 + \left| \frac{\mu_\chi}{2} \chi_i + \frac{f_1'}{3} \epsilon_{ijk} \eta_j' \rho_k' \right|^2 + \left| \frac{\mu_\rho}{2} \rho_i + \frac{f_1'}{3} \epsilon_{ijk} \chi_j' \eta_k' \right|^2 \Bigg]
\end{aligned} \tag{4.3}
$$

The second term is given by

$$
\begin{aligned}
V_D &= -\mathcal{L}_D = \frac{1}{2}(D^a D^a + DD) = \frac{g'^2}{2} \left(-\frac{1}{3} \eta^\dagger \eta + \frac{1}{3} \eta'^\dagger \eta' - \frac{1}{3} \chi^\dagger \chi + \frac{1}{3} \chi'^\dagger \chi' + \frac{2}{3} \rho^\dagger \rho - \frac{2}{3} \rho'^\dagger \rho' \right)^2 \\
&+ \frac{g^2}{8} (\eta_i^\dagger \lambda_{ij}^b \eta_j - \eta_i'^\dagger \lambda_{ij}^{*b} \eta_j' + \chi_i^\dagger \lambda_{ij}^b \chi_j - \chi_i'^\dagger \lambda_{ij}^{*b} \chi_j' + \rho_i^\dagger \lambda_{ij}^b \rho_j - \rho_i'^\dagger \lambda_{ij}^{*b} \rho_j')^2,
\end{aligned} \tag{4.4}
$$

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and

$$
\begin{aligned}
V_{\text{soft}} &= -\mathcal{L}_{\text{soft}} = m_\eta^2 \eta^\dagger \eta + m_\rho^2 \rho^\dagger \rho + m_\chi^2 \chi^\dagger \chi + m_{\eta'}^2 \eta'^\dagger \eta' \\
&+ m_{\rho'}^2 \rho'^\dagger \rho' + m_{\chi'}^2 \chi'^\dagger \chi' - \epsilon_{ijk}(k_1 \rho_i \chi_j \eta_k + k_1' \rho_i' \chi_j' \eta_k' + h.c.).
\end{aligned} \tag{4.5}
$$

For convenience, we rewrite the expansion of the neutral scalar fields:

$$
\eta = \begin{pmatrix} v + \eta_1 + i\eta_2 \\ 0 \\ \eta_3 + i\eta_4 \end{pmatrix}, \quad
\chi = \begin{pmatrix} \chi_1 + i\chi_2 \\ 0 \\ w + \chi_3 + i\chi_4 \end{pmatrix}, \quad
\rho = \begin{pmatrix} 0 \\ u + \rho_1 + i\rho_2 \\ 0 \end{pmatrix} \tag{4.6}
$$

Similarly for the prime fields

$$
\eta' = \begin{pmatrix} v' + \eta_1' + i\eta_2' \\ 0 \\ \eta_3' + i\eta_4' \end{pmatrix}, \quad
\chi' = \begin{pmatrix} \chi_1' + i\chi_2' \\ 0 \\ w' + \chi_3' + i\chi_4' \end{pmatrix}, \quad
\rho' = \begin{pmatrix} 0 \\ u' + \rho_1' + i\rho_2' \\ 0 \end{pmatrix} \tag{4.7}
$$

For the sake of simplicity, here we assume that vacuum expectation values (VEVs) are real. This means that the CP violation through the scalar exchange is not considered in this work.

Then, the real part, of Eqs.(4.6,4.7), $(\eta_1, \eta_1', \chi_1, \chi_1', ...)$ are called CP-even scalar or scalar, while imaginary parts $(\eta_2, \eta_2', \chi_2, \chi_2', ...)$ - CP-odd scalar or pseudoscalar field. In this paper, we call them scalar and pseudoscalar. Returning to Eqs (4.3)-(4.5), by requirement of vanishing the linear terms in fields, we get, in the tree level approximation, the following constraint equations

$$
\begin{aligned}
m_\eta^2 &= -\frac{1}{36v}\left(-6vw^2 g^2 + 4vw^2 f_1^2 + 4vw^2 g'^2 - 6vu^2 g^2 + 4vu^2 f_1^2 - 8vu^2 g'^2 \right. \\
&- 12vv'^2 g^2 - 4vv'^2 g'^2 - 4vw'^2 g'^2 + 6vu'^2 g^2 - 36wuk_1 + 8vu'^2 g'^2 \\
&+ \left. 9v\mu_\eta^2 + 12v^3 g^2 + 2v^3 g'^2 + 6wu'\mu_\rho f_1 + 6uw'\mu_\chi f_1 + 6w'u'\mu_\eta f_1' \right), \\
m_\chi^2 &= -\frac{1}{36w}\left(-36vuk_1 + 6vu'\mu_\rho f_1 - 6v^2 wg^2 + 4v^2 wf_1^2 + 4v^2 wg'^2 + 6v'u'\mu_\chi f_1' \right. \\
&- 6wu^2 g^2 + 4wu^2 f_1^2 - 8wu^2 g'^2 + 6wv'^2 g^2 - 4wv'^2 g'^2 + 6uv'\mu_\eta f_1 \\
&- \left. 12ww'^2 g^2 - 4ww'^2 g'^2 + 6wu'^2 g^2 + 8wu'^2 g'^2 + 9w\mu_\chi^2 + 12w^3 g^2 + 4w^3 g'^2 \right), \\
m_\rho^2 &= -\frac{1}{36u}\left(-36vwk_1 + 6vw'\mu_\chi f_1 - 6v^2 ug^2 + 4v^2 uf_1^2 - 8v^2 ug'^2 + 6wv'\mu_\eta f_1 \right. \\
&- 6w^2 ug^2 + 4w^2 uf_1^2 - 8w^2 ug'^2 + 6uv'^2 g^2 + 8uv'^2 g'^2 + 6uw'^2 g^2 + 16u^3 g'^2 \\
&+ \left. 8uw'^2 g'^2 - 12uu'^2 g^2 - 16uu'^2 g'^2 + 9u\mu_\rho^2 + 12u^3 g^2 + 6v'w'\mu_\rho f_1' \right), \\
m_{\eta'}^2 &= -\frac{1}{36v'}\left(-12v^2 v'g^2 - 4v^2 v'g'^2 + 6wu\mu_\eta f_1 + 6wu'\mu_\chi f_1' + 6w^2 v'g^2 \right. \\
&- 4w^2 v'g'^2 + 6uw'\mu_\rho f_1' + 6u^2 v'g^2 + 8u^2 v'g'^2 - 6v'w'^2 g^2 + 4v'w'^2 (f_1')^2 + 4v'w'^2 g'^2 \\
&- \left. 36w'u'k_1' - 6v'u'^2 g^2 + 4v'u'^2 (f_1')^2 - 8v'u'^2 g'^2 + 9v'\mu_\eta^2 + 12v'^3 g^2 + 4v'^3 g'^2 \right),
\end{aligned}
$$

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$$
\begin{aligned}
m_{\chi'}^2 &= -\frac{1}{36w'}\Big(6vu\mu_\chi f_1 + 6vu'\mu_\eta f_1' - 4v^2w'g'^2 - 12w^2w'g^2 - 4w^2w'g'^2 + 6uv'\mu_\rho f_1' \\
&+ 6u^2w'g^2 + 8u^2w'g'^2 - 36v'u'k_1' - 6v'^2w'g^2 + 4v'^2w'(f_1')^2 + 4v'^2w'g'^2 \\
&- 6w'u'^2g^2 + 4w'u'^2(f_1')^2 - 8w'u'^2g'^2 + 9w'\mu_\chi^2 + 12w'^3g^2 + 4w'^3g'^2\Big), \\
m_{\rho'}^2 &= -\frac{1}{36u'}\Big(6vw\mu_\rho f_1 + 6vw'\mu_\eta f_1' + 6v^2u'g^2 + 8v^2u'g'^2 + 6wv'\mu_\chi f_1' + 6w^2u'g^2 \\
&+ 8w^2u'g'^2 - 12u^2u'g^2 - 16u^2u'g'^2 - 36v'w'k_1' - 6v'^2u'g^2 + 4v'^2u'(f_1')^2 - 8v'^2u'g'^2 \\
&- 6w'^2u'g^2 + 4w'^2u'(f_1')^2 - 8w'^2u'g'^2 + 9u'\mu_\rho^2 + 12u'^3g^2 + 16u'^3g'^2\Big).
\end{aligned}
\tag{4.8}
$$

The most general scalar potential (i.e. the one including all terms consistent with the gauge invariance and renormalizability as considered in [7]) is very complicated. However, we can use one approximation to simplify our analysis. Before writing this approximation, we will analyze the gauge sector.

We can write the gauge mass term as

$$
\begin{aligned}
\mathcal{L}_{Higgs} &= (\mathcal{D}_m\eta)^\dagger(\mathcal{D}^m\eta) + (\mathcal{D}_m\rho)^\dagger(\mathcal{D}^m\rho) + (\mathcal{D}_m\chi)^\dagger(\mathcal{D}^m\chi) + (\overline{\mathcal{D}_m}\eta')^\dagger(\overline{\mathcal{D}^m}\eta') \\
&+ (\overline{\mathcal{D}_m}\rho')^\dagger(\overline{\mathcal{D}^m}\rho') + (\overline{\mathcal{D}_m}\chi')^\dagger(\overline{\mathcal{D}^m}\chi'),
\end{aligned}
\tag{4.9}
$$

where $\mathcal{D}_m$ is the triplet covariant derivative given by

$$
\mathcal{D}_m\phi_i = \partial_m\phi_i - ig\left(\vec{V}_m.\frac{\vec{\lambda}}{2}\right)_i^j\phi_j - ig'N_{\phi_i}V_m'\phi_i,
\tag{4.10}
$$

while $\overline{\mathcal{D}_m}$ is the anti-triplet covariant derivative, and it is written as

$$
\overline{\mathcal{D}_m}\phi_i = \partial_m\phi_i + ig\left(\vec{V}_m.\frac{\vec{\overline{\lambda}}}{2}\right)_i^j\phi_j + ig'N_{\phi_i}V_m'\phi_i,
\tag{4.11}
$$

and $\phi = \eta, \rho, \chi, \eta', \rho', \chi'$.

The non-Hermitian gauge bosons $\sqrt{2}\,W_m^+ = V_m^1 - iV_m^2$, $\sqrt{2}\,Y_m^- = V_m^6 - iV_m^7$, $\sqrt{2}\,X_m^0 = V_m^4 - iV_m^5$ have the following masses [3]:

$$
M_W^2 = \frac{g^2}{4}(U^2 + V^2),\quad M_Y^2 = \frac{g^2}{4}(U^2 + W^2),\quad M_X^2 = \frac{g^2}{4}(V^2 + W^2)
\tag{4.12}
$$

where $V^2 = v^2 + v'^2$, $U^2 = u^2 + u'^2$ and $W^2 = w^2 + w'^2$.

While in the (V_{3m}, V_{8m}, V_m') basis we have the mass square of the real vector bosons given by:

$$
\frac{g^2}{4}\begin{pmatrix}
V^2 + U^2 & \frac{1}{\sqrt{3}}(V^2 - U^2) & -\frac{2t}{3}(V^2 + 2U^2) \\
\frac{1}{\sqrt{3}}(V^2 - U^2) & \frac{1}{3}(V^2 + U^2 + 4W^2) & -\frac{2t}{3\sqrt{3}}(V^2 - 2U^2 - 2W^2) \\
-\frac{2t}{3}(V^2 + 2U^2) & -\frac{2t}{3\sqrt{3}}(V^2 - 2U^2 - 2W^2) & \frac{4t^2}{9}(V^2 + 4U^2 + W^2),
\end{pmatrix}
\tag{4.13}
$$

The eigenstates of Eq. (4.13) are

$$A_m = \frac{\sqrt{3}}{4t^2 + 3} \left(t \, V_{3m} - \frac{t}{\sqrt{3}} V_{8m} + V'_m \right), \qquad (4.14)$$

for the photon, and

$$Z^0_m \approx \frac{3t}{4t^2 + 15t^2 + 9} \left[-\left(\frac{t^2 + 3}{3t} \right) V_{3m} - \frac{t}{\sqrt{3}} V_{8m} + V'_m \right], \qquad (4.15)$$

and

$$Z^{0\prime}_m \approx \frac{t}{t^2 + 3} \left(\frac{\sqrt{3}}{t} V_{8m} + V'_m \right) \qquad (4.16)$$

for the Z^0 and $Z^{0\prime}$, we have neglected the mixing among Z^0 and $Z^{0\prime}$. Their masses are

$$M_A^2 \;\; = \;\; 0 \qquad (4.17)$$

$$M_Z^2 \;\; \simeq \;\; \frac{g^2}{4} \left(\frac{3 + 4t^2}{3 + t^2} \right) (V^2 + U^2) \qquad (4.18)$$

$$M_{Z'}^2 \;\; \simeq \;\; \frac{g^2}{3} \left(1 + \frac{t^2}{3} \right) W^2. \qquad (4.19)$$

so that $M_Z^2 / M_W^2 \approx (3 + 4t^2)/(3 + t^2) = 1/\cos^2\theta_W$, and

$$t^2 = \left(\frac{g'}{g} \right)^2 = \frac{\sin^2\theta_W}{1 - \frac{4}{3}\sin^2\theta_W}. \qquad (4.20)$$

Eq. (4.12) yields the splitting between the bilepton masses

$$|M_X^2 - M_Y^2| \leq M_W^2 \qquad (4.21)$$

which is the same as in the non-supersymmetric version [13].

The Eqs.(4.13,4.12) are very important, because the new gauge bosons must be sufficiently heavy to keep consistency with low energy phenomenology. Due this fact the VEV's satisfy the conditions [7]:

$$w, w' \gg v, v', u, u' \qquad (4.22)$$

which are followed from vanishing coupling constant of singlet fields to the SM gauge bosons. In this paper, we will use this approximation.

The mass matrices, thus, can be calculated, using

$$M_{ij}^2 = \frac{\partial^2 V_{MSUSY331}}{\partial \phi_i \partial \phi_j} \qquad (4.23)$$

and evaluated at the chosen minimum, where $\phi = \eta, \rho, \chi, \eta', \rho', \chi'$.

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4.1 Spectrum in the neutral scalar sector

In the approximation (4.22), the mass square matrix of scalar particles in the base of $(\eta_1, \rho_1, \eta_1', \rho_1', \eta_3, \eta_3', \chi_1, \chi_1', \chi_3, \chi_3')$, after imposing the constraint equation, has the form following

$$M_H^2 = \begin{pmatrix} M_{4\eta\rho}^2 & 0 & 0 & 0 & 0 \\ 0 & m_{\eta_3}^2 & 0 & 0 & 0 \\ 0 & 0 & m_{\eta_3'}^2 & 0 & 0 \\ 0 & 0 & 0 & M_{2\chi_1\chi_1'}^2 & 0 \\ 0 & 0 & 0 & 0 & M_{2\chi_3\chi_3'}^2 \end{pmatrix}, \tag{4.24}$$

where

$$M_{4\eta\rho}^2 = \begin{pmatrix} 0 & m_{\eta_1\rho_1}^2 & 0 & m_{\eta_1\rho_1'}^2 \\ m_{\eta_1\rho_1}^2 & 0 & m_{\eta_1'\rho_1}^2 & 0 \\ 0 & m_{\eta_1'\rho_1}^2 & 0 & m_{\eta_1'\rho_1'}^2 \\ m_{\eta_1\rho_1'}^2 & 0 & m_{\eta_1'\rho_1'}^2 & 0 \end{pmatrix}, \tag{4.25}$$

$$M_{2\chi_1\chi_1'}^2 = \frac{g^2}{2} \begin{pmatrix} w'^2 & -ww' \\ -ww' & w \end{pmatrix}, \tag{4.26}$$

$$M_{2\chi_3\chi_3'}^2 = \frac{2}{9}(g'^2 + 3g^2) \begin{pmatrix} w^2 & -ww' \\ -ww' & w'^2 \end{pmatrix}. \tag{4.27}$$

We see that, at the tree level, η_3, η_3' are eigenstates *already* with masses

$$m_{\eta_3}^2 = \frac{1}{18}w^2\left(9g^2 - 2f_1^2\right), \tag{4.28}$$

$$m_{\eta_3'}^2 = -\frac{1}{2}w^2g^2 + \frac{1}{18}w'^2\left(9g^2 - 2\left(f_1'\right)^2\right). \tag{4.29}$$

We remind that $\eta_3, \eta_3', \chi_1, \chi_1'$ are bilepton, while $\eta_1, \rho_1, \eta_1', \rho_1', \chi_3, \chi_3'$ are pure scalars (without lepton number). From (4.24) it follows that there is no mixing between scalars having different lepton numbers.

From (4.26) and (4.27), it is easily to check that

$$\mathrm{Det}M_{\chi_1,\chi_1'}^2 = \mathrm{Det}M_{\chi_3,\chi_3'}^2 = 0, \tag{4.30}$$

$$\mathrm{Tr}M_{\chi_1,\chi_1'}^2 = \frac{g^2}{2}\left(w^2 + w'^2\right), \tag{4.31}$$

$$\mathrm{Tr}M_{\chi_3,\chi_3'}^2 = \frac{1}{9}(g'^2 + 3g^2)\left(w^2 + w'^2\right). \tag{4.32}$$

Therefore, there are two neutral scalar Goldstone bosons and another massive scalar

$$m_{\zeta_{\chi_1\chi_1'}}^2 = \frac{g^2}{2}\left(w^2 + w'^2\right), \tag{4.33}$$

$$m_{\zeta_{\chi_3\chi_3'}}^2 = \frac{2}{9}(g'^2 + 3g^2)\left(w^2 + w'^2\right) \le m_{\zeta_{\chi_1\chi_1'}}^2. \tag{4.34}$$

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Now, we consider 4×4 mass matrix $M^2_{4\eta\rho}$ of $\eta_1, \rho_1, \eta'_1, \rho'_1$ mixing. The elements of $M^2_{4\eta\rho}$ given at Eq.(4.25) are

$$
\begin{aligned}
m^2_{\eta_1\rho_1} &= \frac{f_1}{6}\mu_\chi w' - k_1 w, \\
m^2_{\eta_1\rho'_1} &= \frac{1}{6}\left(f_1\mu_\rho w + f'_1\mu_\eta w'\right), \\
m^2_{\eta'_1\rho_1} &= \frac{1}{6}\left(f'_1\mu_\rho w' + f_1\mu_\eta w\right), \\
m^2_{\eta'_1\rho'_1} &= \frac{f'_1}{6}\mu_\chi w - k'_1 w'.
\end{aligned}
\tag{4.35}
$$

We want to remind that the parameters $\mu_\eta, \mu_\rho, \mu_\chi, k_1$ and k'_1 have mass dimension, while f_1 and f'_1 are dimensionless, see Eq.(3.5).

Solving the characteristic equation, we have four massive fields with the physical eigenvalues

$$
\begin{aligned}
m^2_{H^0_{1,2}} &= \pm\frac{1}{\sqrt{2}}\sqrt{\left(m^4_{\eta_1\rho_1} + m^4_{\eta_1\rho'_1} + m^4_{\eta'_1\rho_1} + m^4_{\eta'_1\rho'_1} - \sqrt{M}\right)}, \\
m^2_{H^0_{3,4}} &= \pm\frac{1}{\sqrt{2}}\sqrt{\left(m^4_{\eta_1\rho_1} + m^4_{\eta_1\rho'_1} + m^4_{\eta'_1\rho_1} + m^4_{\eta'_1\rho'_1} + \sqrt{M}\right)},
\end{aligned}
\tag{4.36}
$$

where

$$
M = \left(m^4_{\eta_1\rho_1} + m^4_{\eta_1\rho'_1} + m^4_{\eta'_1\rho_1} + m^4_{\eta'_1\rho'_1}\right)^2 - 4\left(m^2_{\eta_1\rho'_1}m^2_{\eta'_1\rho_1} - m^2_{\eta_1\rho_1}m^2_{\eta'_1\rho'_1}\right)^2.
\tag{4.37}
$$

4.2 Spectrum in neutral pseudoscalar sector

In the pseudoscalar sector, after imposing the constraint equation, we have two Goldstone bosonsχ_4, χ'_4. Those other have mixing square matrix in the base of $(\eta_2, \rho_2, \eta'_2, \rho'_2, \eta_4, \eta'_4, \chi_2, \chi'_2)$, as follows

$$
M^2_{PH} = \begin{pmatrix} M^2_{4\eta'\rho'} & 0 & 0 & 0 \\ 0 & m^2_{\eta_4} & 0 & 0 \\ 0 & 0 & m^2_{\eta'_4} & 0 \\ 0 & 0 & 0 & M^2_{2\chi_2\chi'_2} \end{pmatrix}
\tag{4.38}
$$

with

$$
M^2_{4\eta'\rho'} = \begin{pmatrix} 0 & m^2_{\eta_2\rho_2} & 0 & m^2_{\eta_2\rho'_2} \\ m^2_{\eta_2\rho_2} & 0 & m^2_{\eta'_2\rho_2} & 0 \\ 0 & m^2_{\eta'_2\rho_2} & 0 & m^2_{\eta'_2\rho'_2} \\ m^2_{\eta_2\rho'_2} & 0 & m^2_{\eta'_2\rho'_2} & 0 \end{pmatrix},
\tag{4.39}
$$

$$
M^2_{2\chi_2\chi'_2} = \frac{g^2}{2}\begin{pmatrix} w'^2 & ww' \\ ww' & w^2 \end{pmatrix}
\tag{4.40}
$$

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We also see that, the bileptons η_4, η'_4 do not mix with others, and they are physical fields with masses given by

$$
\begin{aligned}
m_{\eta_4}^2 &= m_{\eta_3}^2, && (4.41) \\
m_{\eta'_4}^2 &= m_{\eta'_3}^2. && (4.42)
\end{aligned}
$$

The square mass matrix $M_{\chi_2 \chi'_2}$ satisfies condition (4.32). Thus, it gives us one Goldstone boson and one physical massive field $m_{\zeta_{\chi_2 \chi'_2}}^2$ with mass:

$$
m_{\zeta_{\chi_2 \chi'_2}}^2 = m_{\zeta_{\chi_1 \chi'_1}}^2. \tag{4.43}
$$

From 4×4 mass matrix $M_{4\eta'\rho'}^2$, we get the following matrix elements of Eq.(4.39)

$$
\begin{aligned}
m_{\eta_2 \rho_2}^2 &= -\frac{f_1}{6}\mu_\chi w' + k_1 w = -m_{\eta_1 \rho_1}^2, \\
m_{\eta_2 \rho'_2}^2 &= \frac{1}{6}\left(f_1 \mu_\rho w + f'_1 \mu_\eta w'\right) = m_{\eta_1 \rho'_1}^2, \\
m_{\eta'_2 \rho_2}^2 &= \frac{1}{6}\left(f'_1 \mu_\rho w' + f_1 \mu_\eta w\right) = m_{\eta'_1 \rho_1}^2, \\
m_{\eta'_2 \rho'_2}^2 &= -\frac{f'_1}{6}\mu_\chi w + k'_1 w' = -m_{\eta'_1 \rho'_1}^2. \tag{4.44}
\end{aligned}
$$

four massive pseudoscalar bosons with the same mass as in the scalar sector (4.36).

To conclude this section, we note that, the scalar sector contains Goldstone bosons for Z, Z' and X^0, X^{0*}.

4.3   Spectrum in the charged scalar sector

In the charged sector, the 8×8 mass matrix in the basis of $\eta^-, \rho_1^-, \eta^{'-}, \rho_1^{'-}, \rho_2^-, \rho_2^{'-}, \chi^-, \chi^{'-}$ has form as follows:

$$
M_{charge}^2 = \begin{pmatrix} M_{4c\eta'\rho'}^2 & 0 & 0 & 0 \\ 0 & m_{\rho_2^-}^2 & 0 & 0 \\ 0 & 0 & m_{\rho_2^{'-}}^2 & 0 \\ 0 & 0 & 0 & M_{2\chi^-\chi'^+}^2 \end{pmatrix} \tag{4.45}
$$

with

$$
M_{4c\eta'\rho'}^2 = \begin{pmatrix} 0 & m_{\eta^-\rho_1^+}^2 & 0 & m_{\eta^-\rho_1^{'+}}^2 \\ m_{\eta^+\rho_1^-}^2 & 0 & m_{\eta^{'+}\rho_1^-}^2 & 0 \\ 0 & m_{\eta^{'-}\rho_1^+}^2 & 0 & m_{\eta^{'-}\rho_1^{'+}}^2 \\ m_{\eta^+\rho_1^{'-}}^2 & 0 & m_{\eta^{'+}\rho_1^{'-}}^2 & 0 \end{pmatrix}, \tag{4.46}
$$

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$$m^2_{\eta^- \rho_1^+} = wk_1 - \frac{1}{6}w'\mu_\chi f_1 = -m^2_{\eta_1 \rho_1}, \tag{4.47}$$

$$m^2_{\eta^- \rho_1'^+} = -\frac{1}{6}\left(w\mu_\rho f_1 + w'\mu_\eta f_1'\right) = -m^2_{\eta_1 \rho_1'}, \tag{4.48}$$

$$m^2_{\eta^+ \rho_1^-} = k_1 w - \frac{1}{6}w'\mu_\chi f_1 = m^2_{\eta^- \rho_1^+}, \tag{4.49}$$

$$m^2_{\eta'^+ \rho_1^-} = -\frac{1}{6}\left(w\mu_\eta f_1 + w'\mu_\rho f_1'\right) = -m^2_{\eta_1' \rho_1}, \tag{4.50}$$

$$m^2_{\eta'^- \rho_1^+} = -\frac{1}{6}\left(w\mu_\eta f_1 + w'\mu_\rho f_1'\right) = -m^2_{\eta_1' \rho_1}, \tag{4.51}$$

$$m^2_{\eta'^- \rho_1'^+} = -\frac{1}{6}w\mu_\chi f_1' + w'k_1' = -m^2_{\eta_1' \rho_1'}, \tag{4.52}$$

$$m^2_{\eta^+ \rho_1'^-} = -\frac{1}{6}\left(w\mu_\rho f_1 + w'\mu_\eta f_1'\right) = -m^2_{\eta_1 \rho_1'}, \tag{4.53}$$

$$m^2_{\eta'^+ \rho_1'^-} = -\frac{1}{6}w\mu_\chi f_1' + w'k_1' = -m^2_{\eta_1' \rho_1'}, \tag{4.54}$$

$M^2_{4c\eta'\rho'}$ gives four massive charged scalars having mass as (4.36).

We have other couple ρ_2^+, $\rho_2'^-$ do not mix with others, they are physical fields with masses given by

$$m^2_{\rho_2^-} = \frac{g^2}{2}\left(w^2 - w'^2\right) - \frac{f_1^2 w^2}{9}, \tag{4.55}$$

$$m^2_{\rho_2'^-} = -\frac{g^2}{2}\left(w^2 - w'^2\right) - \frac{f_1'^2 w'^2}{9} = m^2_{\eta_3'} \tag{4.56}$$

The $M^2_{2\chi^- \chi'^+}$

$$M^2_{2\chi^- \chi'^+} = \frac{g^2}{2}\begin{pmatrix} w'^2 & -ww' \\ -ww' & w^2 \end{pmatrix}, \tag{4.57}$$

also gives us one Goldstone bosons and one mass as (4.34).

5 Numerical analysis

We will use below the following set of parameters in the scalar potential [7]:

$$f_1 = 2, \quad f_1' = 10^{-3}, \quad \text{(dimensionless)} \tag{5.1}$$

and

$$k_1 = k_1' = 10, \quad [GeV], \tag{5.2}$$

$$\mu_\eta = \mu_\rho = 10, \quad [GeV], \tag{5.3}$$

$$\mu_\chi = 100, \quad [GeV]. \tag{5.4}$$

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Here we assume that $v = 1000$ [GeV], $v' = 1500$ [GeV].

Diagonalizing the matrices we got the mass eigenstates, and below we present our results.

5.1 Neutral real scalar

The eigenvalues in (GeV) of $M^2_{4\eta\rho}$ given at Eq.(4.36)

$$\sqrt{|m^2_{H^0_{1,2}}|} = 123.2, \tag{5.5}$$

$$\sqrt{|m^2_{H^0_{3,4}}|} = 200.5 \tag{5.6}$$

Note that the values presented above are in agreement with the current 95% CL mass bound on the lightest scalar at MSSM which is 91 GeV [14].

To the case of η_3, η'_3 at Eqs.(4.28,4.29) the eigenvalues in (GeV) are

$$\sqrt{|m^2_{H^0_5}|} = 480.7, \tag{5.7}$$

$$\sqrt{m^2_{H^0_6}} = 516.6, \tag{5.8}$$

respectively. On the case of the eigenvalues of $M^2_{2\chi_1\chi'_1}$ and $M^2_{2\chi_3\chi'_3}$ at Eq.(4.26), we got in (GeV) the following value:

$$\sqrt{m^2_{H^0_7}} = 832.7 \tag{5.9}$$

while to $M^2_{2\chi_3\chi'_3}$ at Eq.(4.27), we got

$$\sqrt{m^2_{H^0_8}} = 968. \tag{5.10}$$

5.2 Neutral imaginary scalar

To the eigenvalues of the matrix $M^2_{4\eta'\rho'}$, given at Eq.(4.39), we got the following values in (GeV)

$$\sqrt{|m^2_{A^0_{1,2}}|} = 120.6, \tag{5.11}$$

$$\sqrt{|m^2_{A^0_{3,4}}|} = 201.1. \tag{5.12}$$

Our values again are in agreement with the mass bound on the lightest pseudoscalar at MSSM which is 91.9 GeV [14].

On the other hand the eigenvalue of the massive pseudoscalar of $M^2_{2\chi_2\chi'_2}$ is given ate Eq.(5.9) while to the case of η_4, η'_4 is given at Eqs.(5.7,5.8).

5.3 Charged sector

To the eigenvalues of the matrix $M^2_{4c\eta'\rho'}$, given at Eq.(4.54), we got the following values in (GeV)

$$\sqrt{|m^2_{H^+_{1,2}}|} = 102, \tag{5.13}$$

$$\sqrt{|m^2_{H^+_{3,4}}|} = 124.2 \tag{5.14}$$

These values presented above are in accordance with the mass bound on the lightest charged scalar at MSSM which is 79.3 GeV [14].

To the case of ρ'^-_2, ρ_2 at Eqs. (4.55,4.56)the eigenvalues in (GeV) are

$$\sqrt{m^2_{H^+_5}} = 516.4, \tag{5.15}$$

$$\sqrt{|m^2_{H^+_6}|} = 843.3, \tag{5.16}$$

respectively. On the other hand the eigenvalue of the massive pseudoscalar of $M^2_{2\chi_2\chi'_2}$ is given at Eq.(5.9).

6 Plots

Using the values giving at Eqs.(5.1÷5.4), we are going only changing w and w' in our analysis. We get the following mass elements matrix to Eq.(4.35)

From the Fig.1 the eigenvalue $\sqrt{m^2_{H^0_1}}$ at $w' = 5000$ GeV is 223.7 GeV (Fig. 2 for $w' = 1$ TeV), while in Fig.3 the same eigenvalue rise to 224 GeV, at the end, considering Fig.4 we get 224.7 GeV. Then, we can conclude that the lightest scalar mass of our model has the upper limit around 230 GeV.

From the Fig.5 the eigenvalue $\sqrt{m^2_{A^0_1}}$ at $w' = 5000$ GeV is 223.7 GeV (Fig. 6 for $w' = 1$ TeV), while in Fig.7 the same eigenvalue rise to 222 GeV, at the end, considering Fig.8 we get 224.7 GeV. Then, we can conclude that the lightest scalar mass of our model has the upper limit around 230 GeV.

From the Fig.9 the eigenvalue $\sqrt{m^2_{H^+_1}}$ at $w' = 5000$ GeV is 223.7 GeV (Fig. 10 for $w' = 1$ TeV), while in Fig.11 the same eigenvalue rise to 224 GeV, at the end, considering Fig.12 we get 224.7 GeV. Then, we can conclude that the lightest scalar mass of our model has the upper limit around 230 GeV.

From all our plots we can obviously see that the scalar mass are given by the Eq.(4.36), as we have mention during this article.

7 Conclusions

On this article we constructed all the spectrum from the scalar sector of the supersymmetric 3-3-1 model with RH neutrinos. We show that there is no mixing between scalars having $L = 0$ and bilepton scalars having $L = 2$. On this model we have six Goldstone bosons: two in neutral sector, three in pseudo-scalar sector and one in charged scalar sector. We analyze also, numerically, the values of the masses of their physical mass spectrum of the scalars. All the scalar sector of our model contain the upper limit of 230 GeV to the mass of the lightest scalar. All these values are in agreement with the lower limit of the SM Higgs boson obtained by LEP.

The scalar sector of the non-supersymmetric 3-3-1 model was studied at [12], while the production of the standard model Higgs boson at pp colliders was studied in Ref.[15]. On this article, if the mass of Z' is not higher than 1 TeV then the process $pp \rightarrow hZ$ is observable at LHC in the case $M_h < 780$ GeV. If $M_{Z'}$ is from 2 until 4 TeV then the process is observable, for $M_h < 600$ GeV. This analyze is still hold if we assume that the sparticles are heavier than the usual ones.

At last the gauge boson production was analyzed [16], and on this article a complete set of quadratic gauge boson coupling in both 3-3-1 models was presented. The authors deduced that at tree level the quartic divergences are canceled and then unitarity is satisfied. This analyze is still hold on this model.

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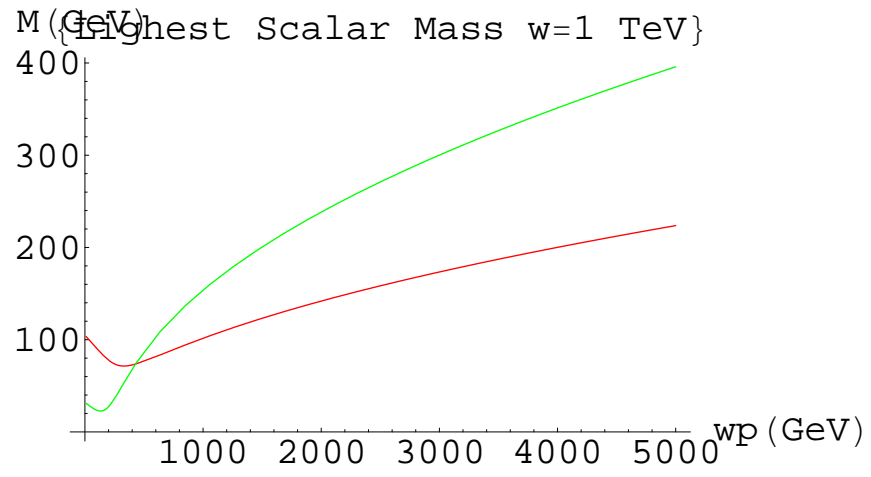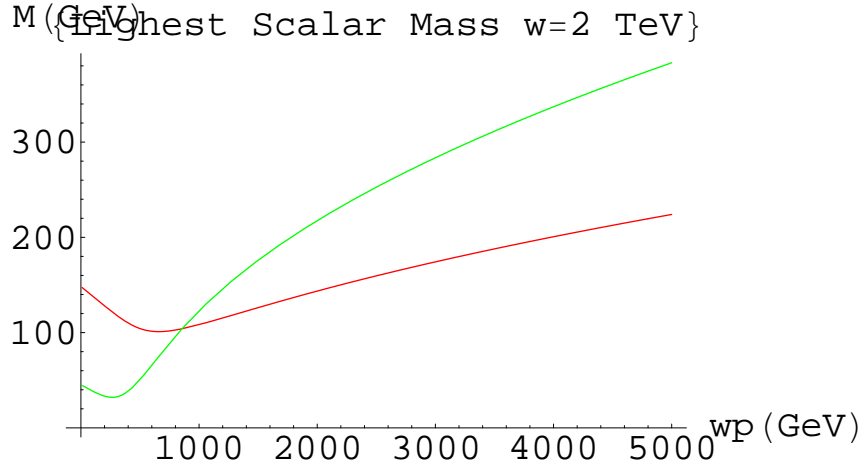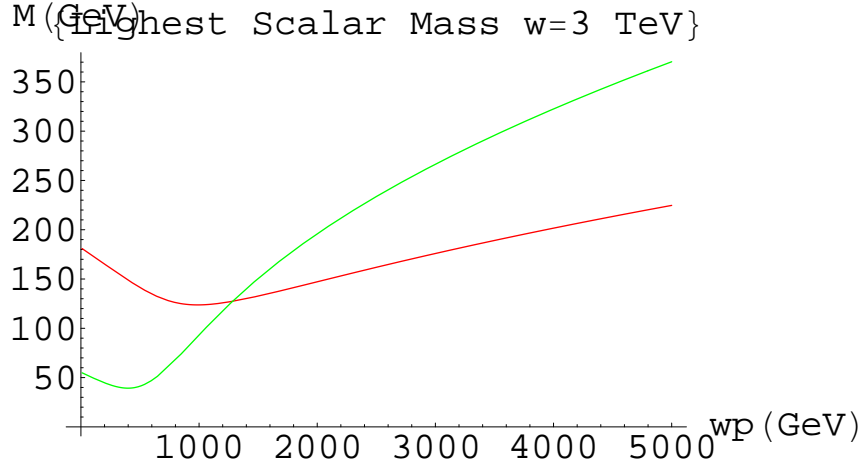
Figure 1: The masses $\sqrt{m^2_{H^0_1}}$ (red lines), and $\sqrt{m^2_{H^0_3}}$ (green lines) of the 4×4 matrix in the neutral scalar as function of w'.
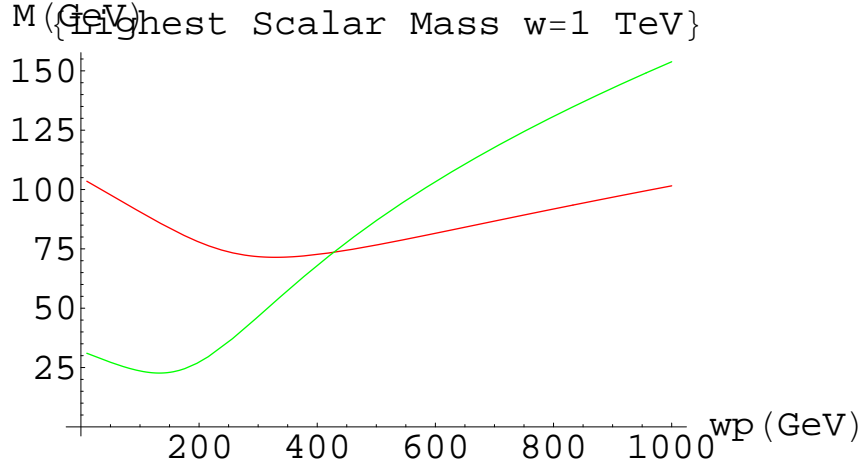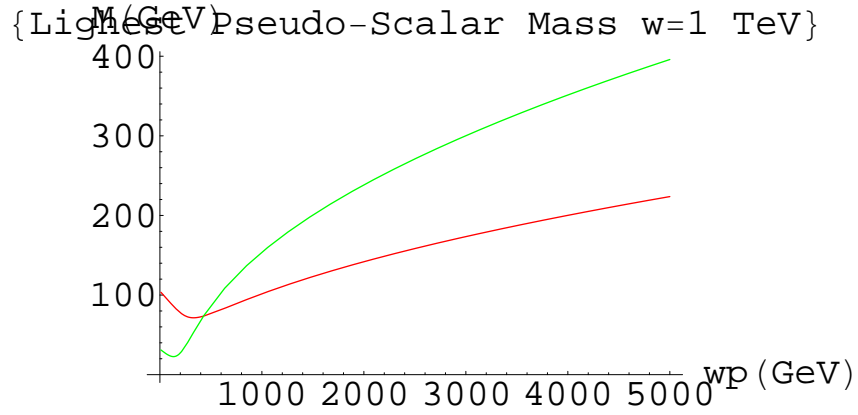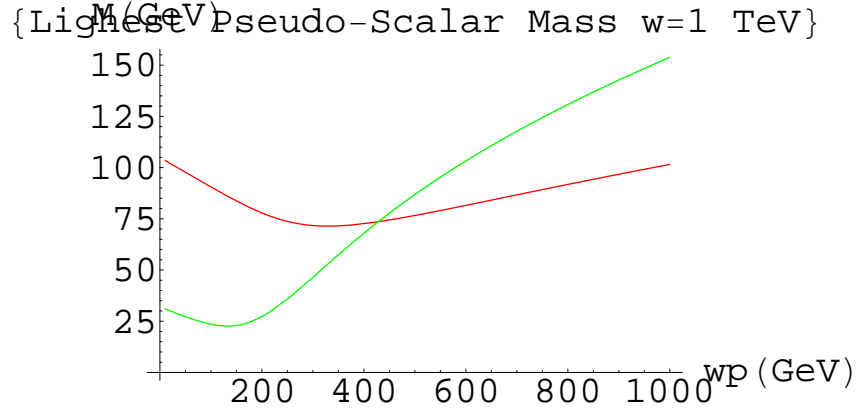
Figure 2: The masses $\sqrt{m^2_{H^0_1}}$ (red lines), and $\sqrt{m^2_{H^0_3}}$ (green lines) of the 4×4 matrix in the charged scalar as function of w'.

Figure 3: The masses $\sqrt{m^2_{H^0_1}}$ (red lines), and $\sqrt{m^2_{H^0_3}}$ (green lines) of the 4×4 matrix in the neutral scalar as function of w'.

Figure 4: The masses $\sqrt{m^2_{H^0_1}}$ (red lines), and $\sqrt{m^2_{H^0_3}}$ (green lines) of the 4×4 matrix in the neutral scalar as function of w'.
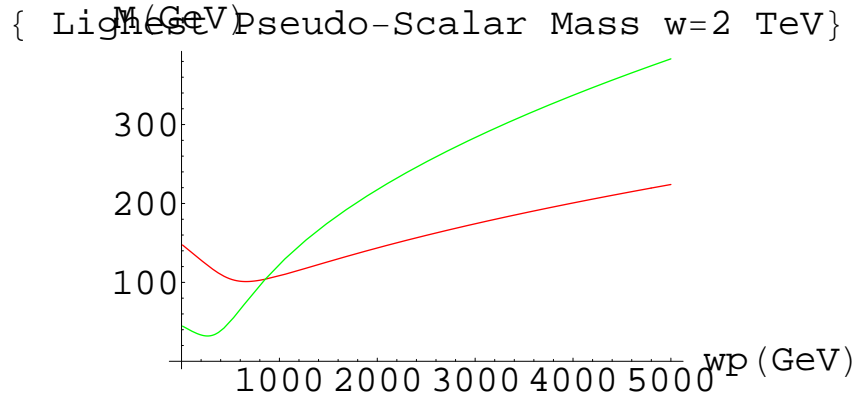

Figure 5: The masses $\sqrt{m^2_{A^0_1}}$ (red lines), and $\sqrt{m^2_{A^0_3}}$ (green lines) of the 4×4 matrix in the neutral pseudo scalar as function of w'.

{Lightest Pseudo-Scalar Mass w=1 TeV}

M(GeV)

150
125
100
75
50
25

200 400 600 800 1000   wp (GeV)

Figure 6: The masses $\sqrt{m^2_{A^0_1}}$ (red lines), and $\sqrt{m^2_{A^0_3}}$ (green lines) of the 4×4 matrix in the charged scalar as function of w'.

{ Lightest Pseudo-Scalar Mass w=2 TeV}

M(GeV)

300

200

100

1000 2000 3000 4000 5000   wp (GeV)

Figure 7: The masses $\sqrt{m^2_{A^0_1}}$ (red lines), and $\sqrt{m^2_{A^0_3}}$ (green lines) of the 4×4 matrix in the neutral pseudo scalar as function of w'.

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Figure 8: The masses $\sqrt{m_{A_1^0}^2}$ (red lines), and $\sqrt{m_{A_3^0}^2}$ (green lines) of the 4×4 matrix in the neutral pseudo scalar as function of w'.
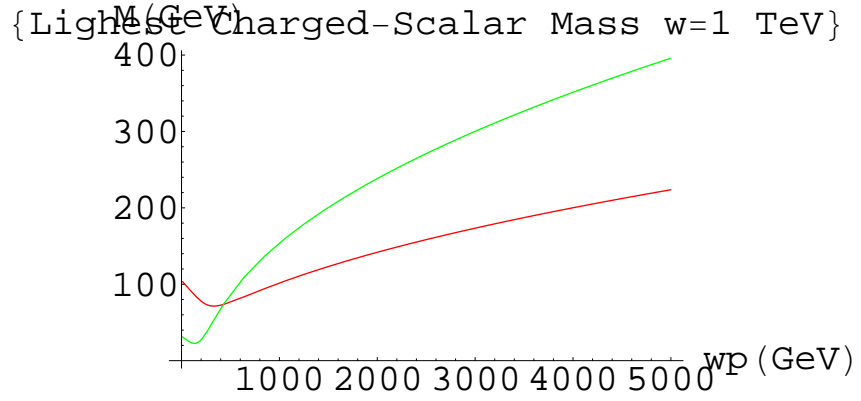

Figure 9: The masses $\sqrt{m_{H_1^+}^2}$ (red lines), and $\sqrt{m_{H_3^+}^2}$ (green lines) of the 4×4 matrix in the charged scalar as function of w'.
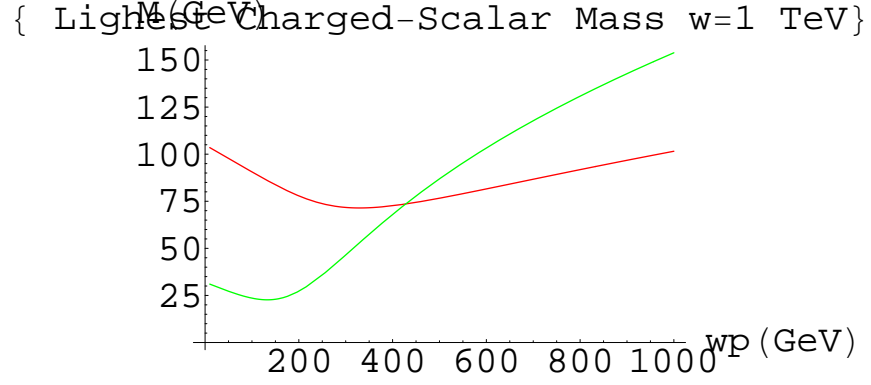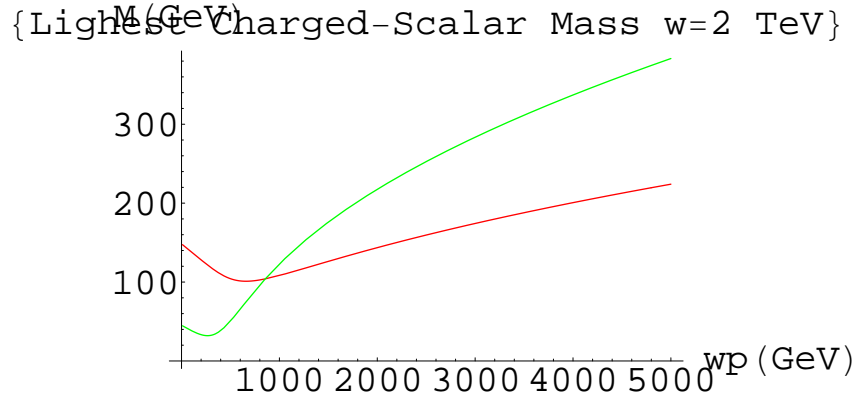
{ Lightest Charged–Scalar Mass w=1 TeV}

M(GeV)

wp(GeV)

Figure 10: The masses $\sqrt{m^2_{H^+_1}}$ (red lines), and $\sqrt{m^2_{H^+_3}}$ (green lines) of the 4×4 matrix in the charged scalar as function of w'.

{Lightest Charged–Scalar Mass w=2 TeV}

M(GeV)

wp(GeV)

Figure 11: The masses $\sqrt{m^2_{H^+_1}}$ (red lines), and $\sqrt{m^2_{H^+_3}}$ (green lines) of the 4×4 matrix in the charged scalar as function of w'.

M(GeV)

{Lightest Charged-Scalar Mass w=3 TeV}

wp(GeV)

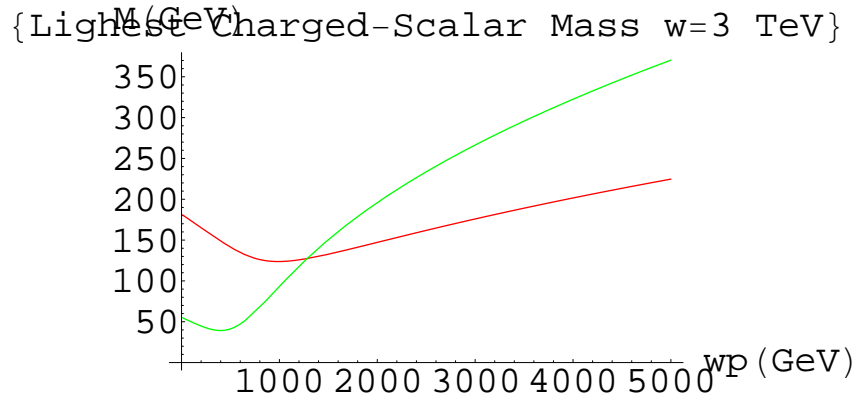Figure 12: The masses $\sqrt{m^2_{H_1^+}}$ (red lines), and $\sqrt{m^2_{H_3^+}}$ (green lines) of the 4×4 matrix in the charged scalar as function of w'.